Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–IV(NEW) – EXAMINATION – SUMMER 2019

Subject Code:2140001 **Date:09/05/2019**

Subject Name: Mathematics-4

Time: 02:30 PM TO 05:30 PM **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** Find the principal argument of $z = \frac{i}{\sqrt{3}+i}$ **03**

(b) Check whether the following functions are analytic or not at any point: **04**

(i) $f(z) = x^2 + ixy$ (ii) $f(z) = z^2$

(c) (i) Expand $f(z) = z\cos\left(\frac{1}{z^3}\right)$ in Laurent's series near $z = 0$ and identify the singularity. **07**

(ii) Show that if c is any n^{th} root of unity other than unity itself, than $1 + c + c^2 + \cdots + c^{n-1} = 0$.

Q.2 **(a)** Find and sketch the image of the region $|z| < 1$ under the transformation $2z - i$. **03**

(b) Show that the function $u(x,y) = y^3 - 3x^2y$ is harmonic in some domain D and find its conjugate $v(x,y)$. **04**

(c) Find the Mobius transformation that maps the points $z = 1, i, -1$ into the points $w = i, 0, -i$. Hence find the image of $|z| = 1$. **07**

OR

(c) Evaluate the integral $\int_C \mathrm{Re}(z^2)\, dz$, where C is the boundary of the square with vertices $0, i, 1+i, 1$ in clockwise direction. **07**

Q.3 **(a)** Evaluate $\int_0^{1+i} (x^2 + iy)\, dz$ along the path $y = x^2$. **03**

(b) Find the residue at each pole of $f(z) = \frac{ze^{iz}}{z^2+9}$ **04**

(c) Expand $f(z) = \frac{1}{(z+1)(z-2)}$ in Laurent's series in the region **07**

(i) $|z| < 1$ (ii) $1 < |z| < 2$ (iii) $|z| > 2$.

OR

Q.3 **(a)** Write the Cauchy integral formula and using it evaluate $\int_C \frac{\cos z}{z+\pi} dz$ where C is the circle $|z| = 4$. **03**

(b) Evaluate $\oint_C \frac{2z-1}{z(z+1)(z-3)} dz$, where C is the circle $|z| = 2$. **04**

(c) Using the residue theorem, evaluate $\int_0^{2\pi} \frac{d\theta}{5 - 3\sin\theta}$ **07**

Q.4 **(a)** Find the positive root of the equation $2\sin x - x = 0$ using bisection method in six stages. **03**

(b) Solve the following system of equations by Gauss Seidel method: **04**

$28x + 4y - z = 32 \qquad 2x + 17y + 4z = 35 \qquad x + 3y + 10z = 24$

Correct up to two decimal places.

(c) Using the power method find the largest eigenvalue of the matrix **07**

$$\begin{bmatrix} 1 & -3 & 2 \\ 4 & 4 & -1 \\ 6 & 3 & 5 \end{bmatrix}$$

OR

Q.4 **(a)** Use the secant method in three stages to find the root of the equation $\cos x - xe^x = 0$. **03**

(b) Find an approximate value of $f(3.6)$ using Newton's backward difference formula from the following data: **04**

x	0	1	2	3	4
f(x)	-5	1	9	25	55

(c) Using Lagrange's interpolation formula find *y* when *x* = 5 from the following table: **07**

x	1	2	3	4	7
y	2	4	8	16	128

Q.5 **(a)** Use Simpson's 1/3 rule to evaluate $\int_1^2 e^{-\frac{x}{2}} dx$. Take h = 0.25. **03**

(b) Use Gauss elimination method to solve the system of equations **04**

$2x_1 + 4x_2 - 6x_3 = -4; \qquad x_1 + 5x_2 + 3x_3 = 10; \qquad x_1 + 3x_2 + 2x_3 = 5.$

(c) Derive Euler's formula to solve the initial value problem $\frac{dy}{dx} = f(x, y);\ \ y(x_0) = y_0$. Find $y(0.1)$ for $\frac{dy}{dx} = x^2 + y$, where $y(0) = 1$ using improved Euler's method. Take $h = 0.05$. **07**

OR

Q.5 **(a)** Find the real root of the equation $x^3 - 9x + 1 = 0$ up to five decimal places by the Newton-Raphson's method. Take $x_0 = 3$. **03**

(b) Find $f(15)$ from the following table using Newton's divided difference formula: **04**

x	4	5	7	10	11	13
f(x)	48	100	204	900	1210	2028

(c) Apply fourth order Runge-Kutta method to find $y(0.1)$ and $y(0.2)$ for the differential equation $\frac{dy}{dx} = 3x + \frac{1}{2}y,\ \ y(0) = 1$. Take $h = 0.1$. **07**

****